

CMPUT 350 F2023
A2 Problem 2 Solutions

a) Claim: $h^*(n)$ is consistent.

Proof:

Need to show:

1) $h^*(v) \leq h^*(n) + \text{cost}(v,n)$ for arbitrary node v with neighbour n

If $h^*(v) > h^*(n) + \text{cost}(v,n)$, then there is a path $v \rightarrow n \rightarrow \dots G$ which with length $< h^*(v)$. This means that $h^*(v)$ is not minimal - a contradiction.

2) $h^*(G) = 0$ for goal node G

This is true because no steps need to be taken to arrive at a goal

QED

b) We need to show: $\max(h_1(n), \dots, h_k(n)) \leq h^*(n)$

L.H.S. $\leq \max(h^*(n), \dots, h^*(n)) = h^*(n) \leq h^*(n)$

$\begin{array}{c} | \quad ||| \quad | \\ h_i(n) \text{ admissible} \end{array}$

QED

c) For node v and neighbour n we have:

$h(v) := \max(h_1(v), \dots, h_k(v)) \leq \max(h_1(n) + \text{cost}(v,n), \dots, h_k(n) + \text{cost}(v,n))$

$\begin{array}{c} | \\ h_i \text{ consistent} \end{array}$

$\begin{aligned} &= \max(h_1(n), \dots, h_k(n)) + \text{cost}(v,n) \\ &= h(n) + \text{cost}(v,n) \end{aligned}$

$\Rightarrow h(n)$ is consistent

QED

d) True for $k=1$, but false for $k > 1$. For instance $h_i(n) = h^*(n)$, which is admissible.

Then $h_1(n) + \dots + h_k(n) = k \cdot h^*(n)$ which is not $\leq h^*(n)$ for $k > 1$ QED

e) True. $(h_1(n) + \dots + h_k(n))/k \leq (h^*(n) + \dots + h^*(n))/k = h^*(n)$

$\begin{array}{c} | \\ h_i(n) \text{ admissible} \end{array}$

QED