## On statistical tests

- 1. Competing hypotheses  $H_0$  and  $H_a$ , which are the logical complement of each other. We try to prove  $H_a$  by disproving  $H_0$ .
- 2. The errors. Since sample data is used to make a decision about the entire population, errors can not be avoided with 100% certainty.

type I error – the error of rejecting  $H_0$  even though  $H_0$  is true

type II error – the error of failing to reject  $H_0$  even though  $H_0$  is false

		Truth	
		$H_0$ is true	$H_0$ is false
Test	reject $H_0$	type I error	OK
	do not reject $H_0$	OK	type II error

In a statistical test the probability for an error of type I is controlled ( $\leq \alpha$ ), but the probability for an error of type II should be considered unknown. Therefore,

- when  $H_0$  is rejected, one could make an error of type I, the probability is small, we can be pretty certain the decision is correct (not 100% though).
- when  $H_0$  is not rejected, one could make an error of type II, the probability for this possible error is unknown, better do not commit to the decision (do not accept  $H_0$ ).
- 3. The required components in a complete test
  - (1) Hypotheses and significance level,  $\alpha$ . Choose  $H_0$  and  $H_a$  as logical complements of each other. The significance level is the largest acceptable probability for an error of type I.
  - (2) Assumptions, stating and checking (if possible).
  - (3) Test statistic, obtain the data and find test statistic, including degrees of freedom.
  - (4) P-value. The P-value measures how likely we would see the sample data (or something more extreme) if  $H_0$  would be true. Therefore a small P-value delivers a contradiction to the assumption that  $H_0$  is true, then  $H_a$  as the logical complement must be true.
  - (5) Decision (reject  $H_0$ /do not reject  $H_0$ ).
  - (6) Context, now interpret what the decision means in the context of the problem.