On Random Vectors

A random vector $\vec{Y} = (Y_1, \ldots, Y_n)'$ is said to have a multivariate normal distribution, iff every linear combination of its components $Y = a_1Y_1 + \cdots + a_kY_k$ is normally distributed. That is, for any constant vector $\vec{a} \in \mathbb{R}^n$, the random variable $Y = \vec{a}'\vec{Y}$ has a univariate normal distribution.

Let $\vec{Y}$ be a random vector with mean $E(\vec{Y}) = \vec{\mu}$ and covariance matrix $Cov(\vec{Y}) = C$.

With $\vec{Y}' = (Y_1, Y_2, \ldots, Y_n)$, $\mu' = (\mu_1, \mu_2, \ldots, \mu_n)$, and $C = (c_{ij})_{1 \le i,j \le n}$.
Then

- $E(Y_i) = \mu_i$, $1 \le i \le n$
- $Var(Y_i) = E(Y_i - \mu_i)^2 = c_{ii}$, $1 \le i \le n$
- $Cov(Y_i, Y_j) = E(Y_i - \mu_i)(Y_j - \mu_j) = c_{ij}$, $1 \le i, j \le n$

If Y_i and Y_j are independent then $Cov(Y_i, Y_j) = 0$ (for multivariate normal vectors the reverse is true as well).

Let $A \in R^{m \times n}$, then

- $E(A\vec{Y}) = A\ E(\vec{Y}) = A\vec{\mu}$
- $\mathrm{Cov}(A\vec{Y}) = A\mathrm{Cov}(\vec{Y})A'$
- $A \in R^{n \times n}$ $E(\vec{Y}'A\vec{Y}) = tr(A\mathrm{Cov}(\vec{Y})) + E(\vec{Y})'AE(\vec{Y})$

Let $\vec{h}, \vec{l} \in R^n$, then $\mathrm{Cov}(\vec{l}'\vec{Y}, \vec{h}'\vec{Y}) = l'\mathrm{Cov}(Y)\vec{h}$