

On Mean, Variance and Covariance

Let $X, Y, Z, X_1, X_2, \dots, X_n$ be random variable in $\mathbb{R}$ and $c, d \in \mathbb{R}$.

Rules for the mean of random variables.

1. $E(c) = c$.
2. $E(cX) = cE(X)$.
3. $E(X + Y) = E(X) + E(Y)$

Rules for the variance of random variables.

1. $\text{Var}(X) = E[X - E(X)]^2$
2. $\text{Var}(c) = 0$.
3. $\text{Var}(cX) = c^2\text{Var}(X)$.
4. $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$.

Rules for the covariance for pairs of random variables.

1. $\text{Cov}(X, Y) = E[(X - E(X))(Y - E(Y))]$
2. X, Y independent then $\text{Cov}(X, Y) = 0$.
3. $\text{Cov}(X, Y) = \text{Cov}(Y, X)$.
4. $\text{Cov}(X + c, Y + d) = \text{Cov}(X, Y)$.
5. $\text{Cov}(cX, dY) = cd\text{Cov}(X, Y)$.
6. $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z)$.
7. $\text{Cov}(X, X) = \text{Var}(X)$.
8. $\text{Cov}(\sum_{i=1}^n X_i, Y + d) = \sum_{i=1}^n \text{Cov}(X_i, Y)$.