On Linear Algebra

1. For matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times k}$

$$(AB)' = B'A'$$

2. $A \in \mathbb{R}^{k\times k}$ is called symmetric if A'=A.

3. $A \in \mathbb{R}^{k\times k}$ is called idempotent if AA=A.

4. If $A \in \mathbb{R}^{k\times k}$ is idempotent and symmetric then $I - A$ is also idempotent and symmetric.

5. For $A \in \mathbb{R}^{k\times k}$ the trace of A, $tr(A)$, is defined as

$$tr(A) = \sum_{i=1}^{n} a_{ii}$$

   (a) For matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times m}$

$$tr(AB) = tr(BA)$$

   (b) For matrices $A, B \in \mathbb{R}^{k\times k}$ and $c, d \in \mathbb{R}$

$$tr(cA + dB) = c\ tr(A) + d\ tr(B)$$

6. If $A \in \mathbb{R}^{k\times k}$ idempotent, then $tr(A) = rk(A)$.

7. $A \in \mathbb{R}^{k\times k}$ is invertible, if it has rank k.

8. If A is invertible then its inverse, A^{-1}, exists with $AA^{-1} = A^{-1}A = I_k$.

9. If A has full rank and $\vec{b} \in \mathbb{R}^k$ then the system of linear equations, $A\vec{x} = \vec{b}$, has solution $\vec{x} = A^{-1}\vec{b}$.

10. $A \in \mathbb{R}^{k\times k}$ is called orthonormal if $A'A$=I. Which means $A^{-1} = A'$.