On Distributions 2

Let $A \in R^{k \times k}$, $\vec{Y}$ a k dimensional multivariate normal random vector with mean $\vec{\mu}$ and regular covariance matric V.

Let U be a quadratic form $U = \vec{Y}'A\vec{Y}$, then

1. If AV or VA is idempotent with rank p, the U is χ^2 distributed with $df = p$ and non centrality parameter $\lambda = \vec{\mu}'A\vec{\mu}$, write $U \sim \chi^2_{p,\lambda}$.

   If $V = \sigma^2 I_k$ and A idempotent $U/\sigma^2 \sim \chi^2_{p,\lambda}$, with $\lambda = \vec{\mu}'A\vec{\mu}/\sigma^2$

2. Let $B \in R^{q \times k}$ and W the linear form $W = B\vec{Y}$. Then U and W are independent if

$$BVA = 0.$$

3. Let $B \in R^{k \times k}$ and W the quadratic form $W = \vec{Y}'B\vec{Y}$. Then U and W are independent if

$$AVB = 0.$$