due date

Theorem:

$[A|\vec{b}]$ is row equivalent to $[S|\vec{c}]$ in REF

(i) the system is inconsistent if some row of $[S|\vec{c}]$ is of the form

$$[0\ 0 \cdots 0 \,|\, c] \quad c \neq 0$$

(ii) If the number of pivots in S is equal to the number of variables then the system has a unique solution.

(iii) If the number of pivots is smaller than n then the system has infinite solutions

Finish example:

Reduced Row Echelon Form
(RREF)

(i) it is in row echelon form

(ii) all leading entries are one.

(iii) Columns with a leading 1, all other entries are 0.

Gauss - Jordan - Algorithm

1) Gaussian Algorithm $\to$ REF

2) Subtract multiple of bottom row of rows above to create zeros.

Theorem: For a given matrix A there is a unique matrix in RREF, which is row equivalent to A.

Independent?

Gauss

$$\begin{matrix} 2 & 0 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \\ 3 & 6 & 13 \end{matrix} \qquad \begin{matrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \\ 3 & 6 & 13 \end{matrix} \qquad \begin{matrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 4 & 8 \\ 0 & 6 & 10 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 4 & 8 \\ 0 & 6 & 10 \end{matrix} \qquad \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{matrix} \qquad \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 4 \end{matrix} \qquad \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix}$$

Gauss-Jordan

$R_2 - R_3$

$R_1 - R_3$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} \quad \text{RREF}$$

Def:

The rank of a matrix is the number of leading ones in its RREF

$\rightarrow \operatorname{rank}(A) = 3$

Wednesday

Theorem:

Let $[A|\vec{b}]$ be a system of m linear eq. in n variables

(1) The system is consistent if and only if the rank of A and $[A|\vec{b}]$ are equal.

(2) If the system is consistent then the number of parameters in the standard solution is

$$\#\text{ of parameters} = n - \text{rank}(A)$$

$[A|\vec{b}]$ lin system m eq.
n var.

then $[A|\vec{b}]$ is consistent for <u>all</u> $\vec{b}$ if rank $(A) = m$.

Homogeneous System

~~Ax~~ $\quad \vec{b} = \vec{0}$

→ trivial solution

→ do not need to augment the matrix for finding the solution.

$$x_1 + x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 5x_3 = 0$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 5 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \quad x_3 = 0$$

$$x_2 = t, \quad t \in \mathbb{R}$$

$$x_1 = -t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$