Grant MacEwan College

Linear Algebra I – MATH 125
Practise problems for the Final Exam

1. Give the definition of a subspace.

2. Give the definition of the inverse of a matrix.

3. Is the set $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leqslant 1\}$ a subspace of $\mathbb{R}^2$? Justify your answer.

4. Prove: If $\mathbf{u}, \mathbf{v}$ are orthogonal vectors in $\mathbb{R}^n$ then $||\mathbf{u} + \mathbf{v}||^2 = ||\mathbf{u}||^2 + ||\mathbf{v}||^2$

5. Prove: If V is a vector space and $\mathbf{v} \in V$ then $0\mathbf{v} = \mathbf{0}$.

6. Let
$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 0 & 4 & 2 \\ 0 & -1 & 1 & 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
   (a) Give a row-echelon form of matrix A.
   (b) Determine the rank of matrix A.
   (c) Is the linear system $A\mathbf{x} = \mathbf{b}$ consistent? Explain.
   (d) Give a basis of the column space of A.
   (e) Give a basis of the row space.
   (f) Give a basis of the null space.

7. Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
   (a) Find the inverse of A
   (b) Give the determinant of A
   (c) Use Cramer's Rule to find the value of the third component of the solution of $A\mathbf{x} = \mathbf{b}$

8. Let
$$\mathbf{u} = \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$
   (a) Find the plane parallel to $\mathbf{u}$ and $\mathbf{v}$ through the origin.
   (b) Let θ be the vector between $\mathbf{u}$ and $\mathbf{v}$, find $cos(\theta)$.
   (c) Give the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

(d) Give the line orthogonal to the plane from part (a) through point $P(4, -2, 2)$

9. Let

$$P_1 = (-1, 2, 2), P_2 = (2, 4, 0), P_3 = (-1, 3, -2)$$

   (a) Give the area of the triangle determined by points P_1, P_2, and P_3.

   (b) Find the distance between point P_1 and the plane given by $x - y + 3z - 2 = 0$.

10. Simplify $(\mathbf{u} + k\mathbf{v}) \times (\mathbf{u} - l\mathbf{v})$

11. (a) Give the standard matrix of the linear transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}^3$ which reflects a point on the line xy-plane

   (b) Give the standard matrix of the linear transformation $R : \mathbb{R}^3 \mapsto \mathbb{R}^3$ which contracts a vector by a factor of $1/2$.

   (c) Give the standard matrix of the linear transformation $T \circ R : \mathbb{R}^3 \mapsto \mathbb{R}^3$.

   (d) Find $T \circ R(2, 4, -3)$

12. Let

$$\mathbf{v_1} = \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$$

   Let $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$

   (a) Find a basis of $span(S)$.

   (b) Does the set S form a basis of $\mathbb{R}^3$? Explain.

13. True/False, justify your answer.

   (a) If the row echelon form of the augmented matrix for a linear system has a row of zeros then the linear system has infinite many solutions.

   (b) For a $m \times n$ matrix A the matrix AA^T is symmetric.

   (c) Let A, B be square matrices of the same size then $(AB)^2 = A^2B^2$.